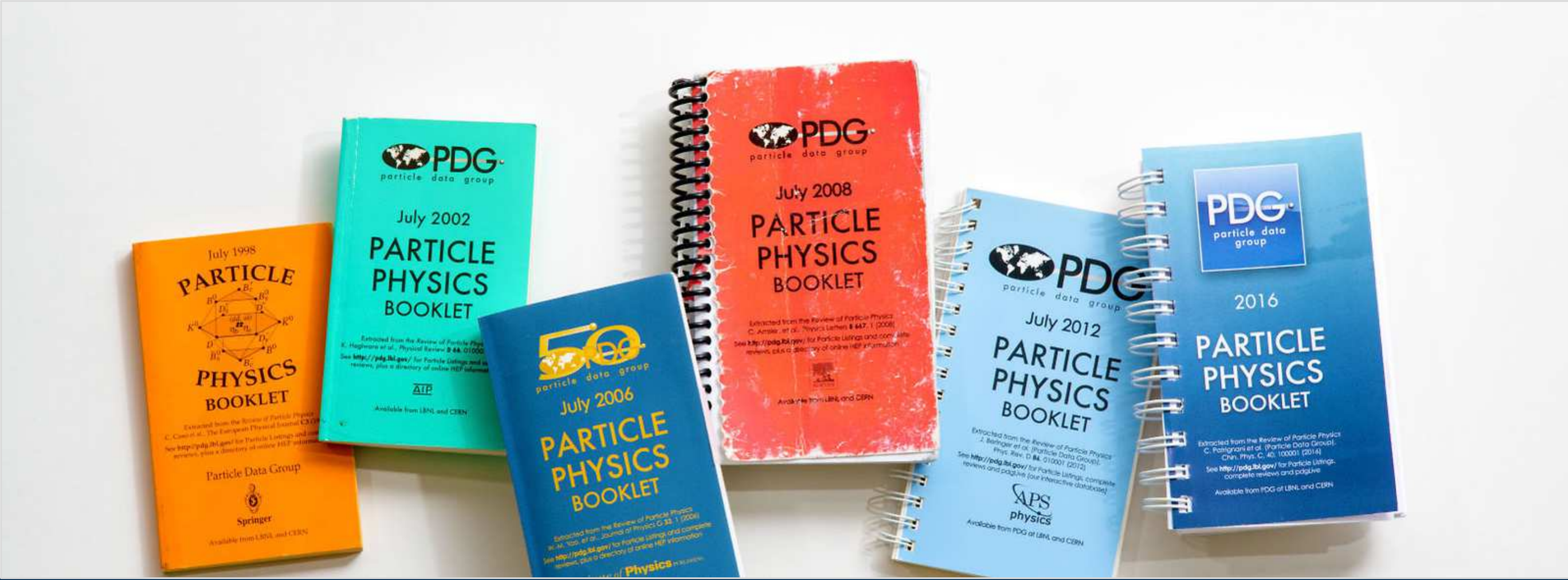


ELEMENTARY PARTICLE PHYSICS

FORCES OF NATURE – FUNDAMENTAL INTERACTIONS (PART II) - QED

MAY 2020 I HANS STRÖHER (FZ JÜLICH, UNIVERSITY OF COLOGNE)

JÜLICH
Forschungszentrum

- Outline:
  - A brief **introduction** (history …)
  - The **tools** (accelerators, targets, detectors … kinematics, …)
  - The **particles** (hadrons, baryons, mesons …)
  - The ***fundamental*** **particles** (quarks, leptons)
  - The **forces** (gravitation, nuclear, weak forces)
  - The ***fundamental*** **interactions** (strong and electro-weak IA)
  - The **Standard Model** of EPP
  - Physics **Beyond the Standard Model** (BSM)
  - Spin-offs – **Applications** of EPP

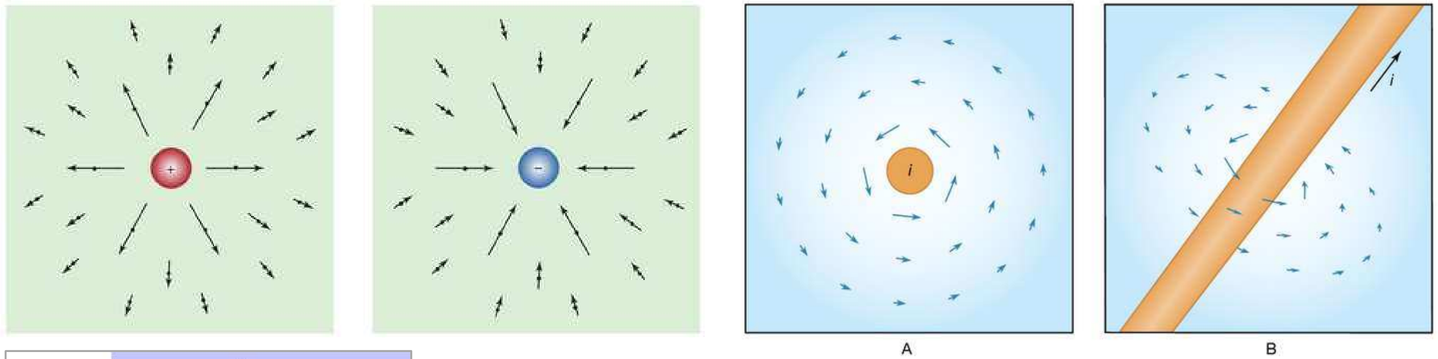
FUNDAMENTAL INTERACTIONS – QED

Prelude

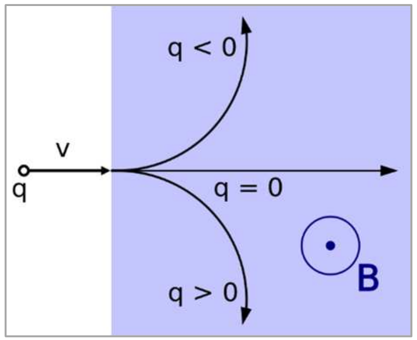
History – the genesis of QED (I)

Classical Electrodynamics provides a description of electromagnetic phenomena whenever the relevant **length scales** and **field strengths** are large enough that quantum mechanical effects are negligible:

Fields:



Forces:



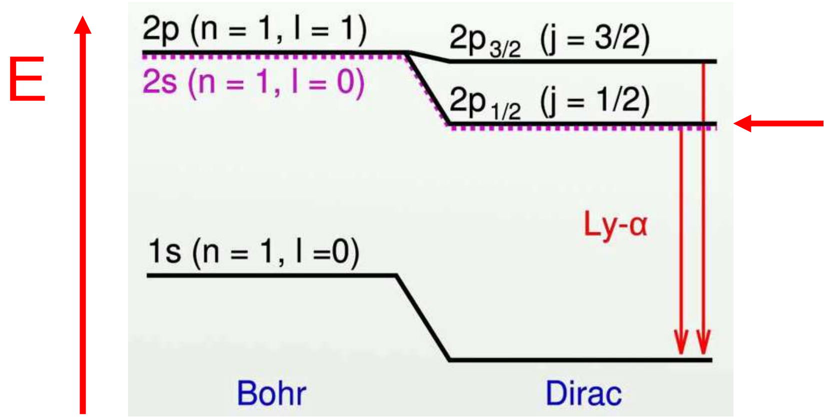
→ **Maxwell equations**, em waves …

FUNDAMENTAL INTERACTIONS – QED

Prelude

History – the genesis of QED (II)

After the invention of quantum mechanics, **P. A. M. Dirac** derived an equation, which describes massive spin-½ particles (e.g. electrons) in a way consistent with both the principles of **quantum mechanics** and **special relativity** → It was validated by accounting for the fine details of the **energy levels** of the **hydrogen atom** in a completely rigorous way:



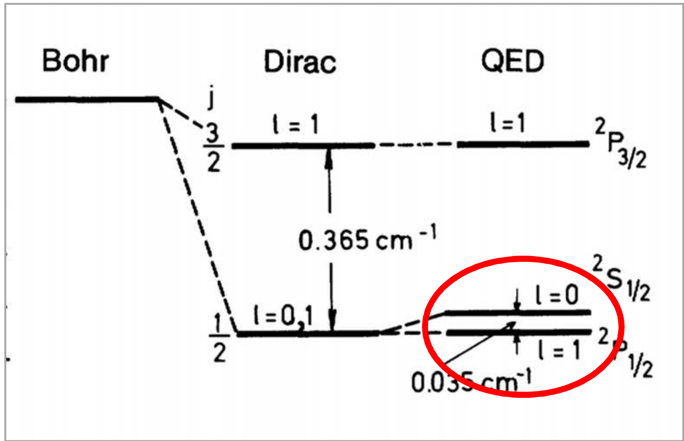
In **Dirac-theory**, the $2s_{1/2}$ and $2p_{1/2}$ energy levels in the hydrogen atom should have the same energies. However, in 1947, **W.E. Lamb** and **R.C. Retherford** measured a tiny shift.

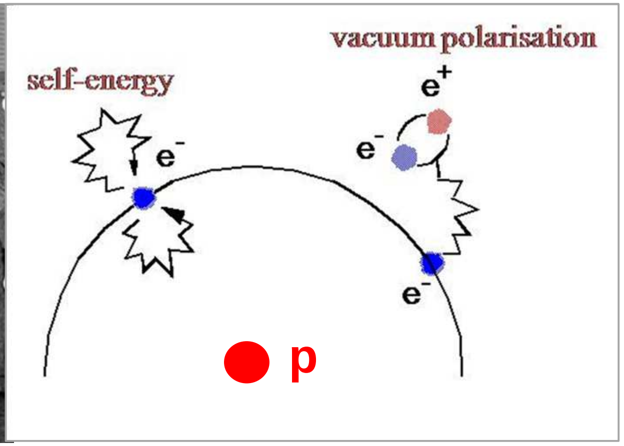
FUNDAMENTAL INTERACTIONS – QED

Prelude

History – the genesis of QED (III)

The so called “**Lamb shift**” was first presented at the Shelter-Island Mtg. (Long Island, USA); many attending theorists argued that the Lamb shift was a result of “**loop diagrams**” of **QED** (→ problem: divergent!))





This led to the theoretical technique of “**renormalization**”

FUNDAMENTAL INTERACTIONS – QED

Prelude

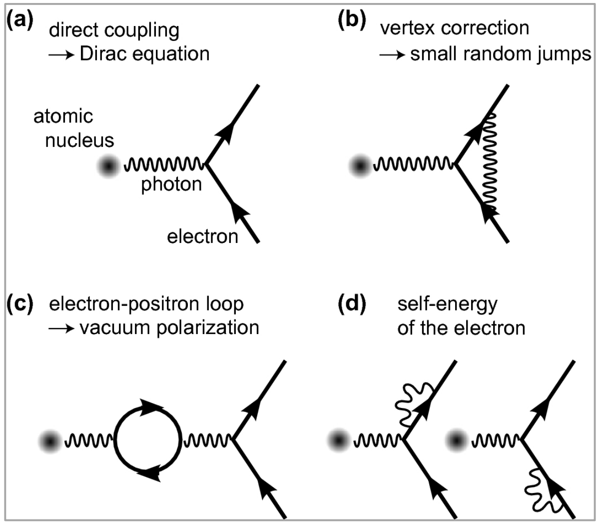
History – the genesis of QED (IV)

When developing quantum electrodynamics, it was discovered that in perturbative calculations **many integrals are divergent**:

(b) an electron emits a photon, emits a second photon, and reabsorbs the first. This process is called a **vertex renormalization.**

(c) a photon creates a virtual electron-positron pair which then annihilate, this is a **vacuum polarization** diagram;

(d) an electron which quickly emits and reabsorbs a virtual photon, called a **self-energy**;



"**Renormalization**" gives finite and physically sensible results by absorbing the divergences into redefinitions of physical quantities.

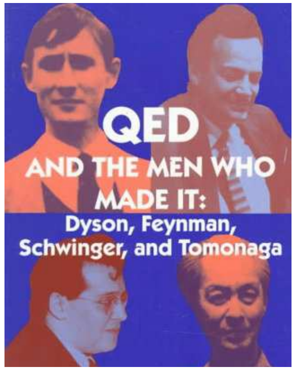
FUNDAMENTAL INTERACTIONS – QED

Prelude

History – the genesis of QED (V)

Quantum electrodynamics (QED) is the relativistic quantum field theory of electrodynamics. It describes **how light and matter interact** and is the first theory in full agreement between quantum mechanics and special relativity

QED gives fantastically **accurate results/predictions** (see below) and has **served as the model** and template for all subsequent quantum field theories (e.g. QCD):



FUNDAMENTAL INTERACTIONS – QED

Basics

Facts – the strength of the interaction (I)

The **fine-structure constant**, commonly denoted α, is a fundamental physical constant, namely the coupling constant **characterizing the strength** of the electromagnetic interaction; **proportional to e^2**:

$$\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c}$$

introduced first by **A. Sommerfeld** to explain the fine structure of the spectral lines in the hydrogen atom

The value of α is: 7.297… $\times 10^{-3}$ (about 1/137); often **1/α** is given:

$$\begin{aligned}\alpha^{-1} &= 137.035\,999\,084\,(33)\,(39) \quad [0.24 \text{ ppb}][0.28 \text{ ppb}],\\ &= 137.035\,999\,084\,(51) \quad [0.37 \text{ ppb}].\end{aligned}$$

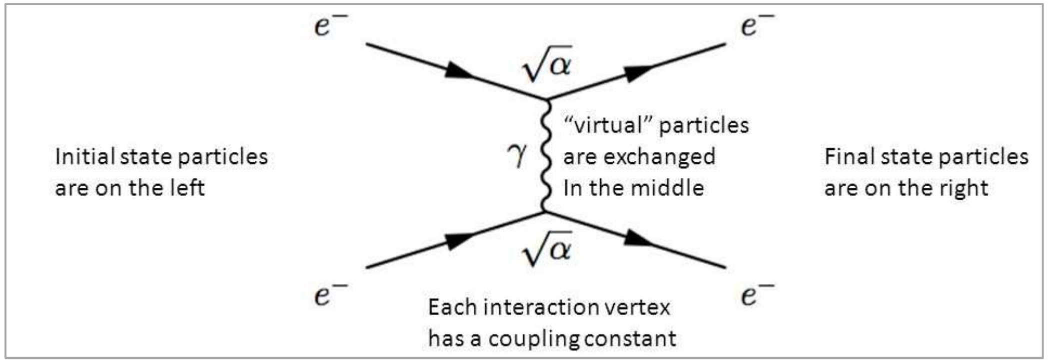
FUNDAMENTAL INTERACTIONS – QED

Basics

Facts – the strength of the interaction (II)

The probability (**cross section σ**) of an electromagnetic reaction can be calculated from drawings of the interaction (Feynman diagram):

Example:



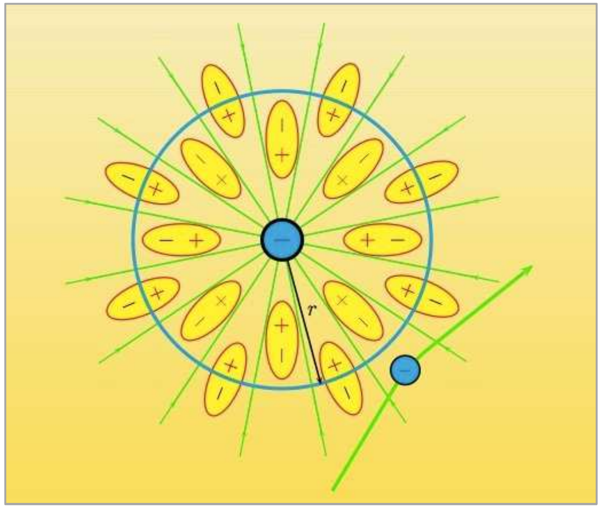
Here, the **reaction amplitude A** is proportional to α (e^2) and the **cross section σ** is proportional to A^2, thus: $\sigma \sim \alpha^2 \sim e^4$

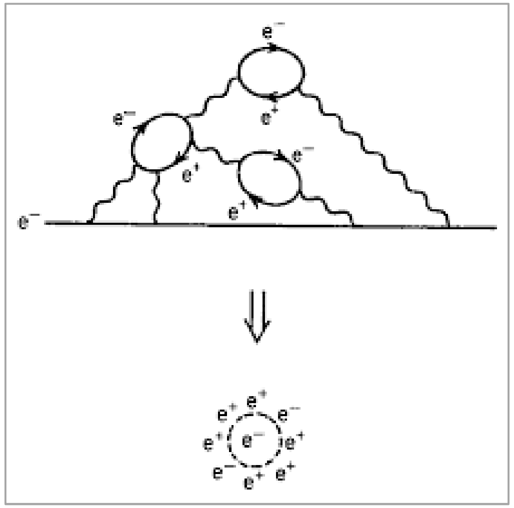
FUNDAMENTAL INTERACTIONS – QED

Basics

Facts – the strength of the interaction (III)

The electromagnetic coupling constant α **is not constant**, but changing ("running"): this is e.g. due to the **polarization of the vacuum** around a ("bare") charge:



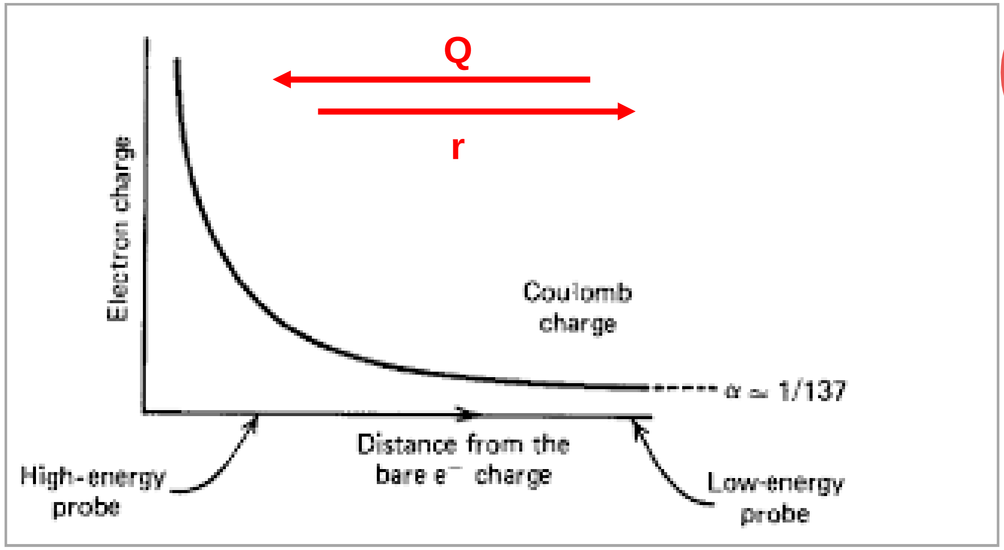


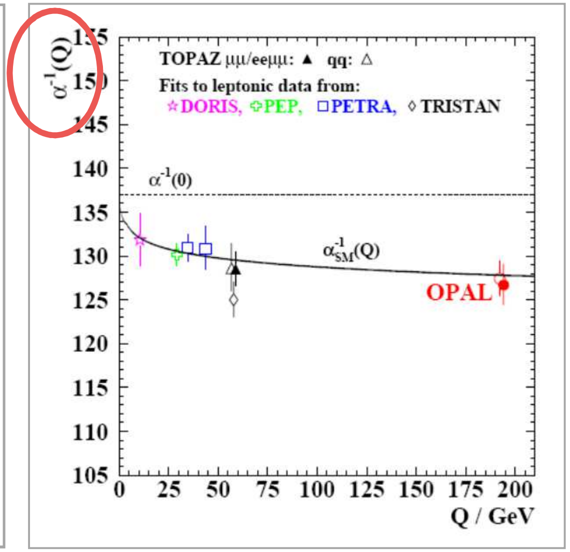
FUNDAMENTAL INTERACTIONS – QED

Basics

Facts – the strength of the interaction (IV)

The electromagnetic coupling constant α **increases** as function of **momentum transfer Q** (the closer one gets to the bare charge), i.e. **decreases with distance r** from it:





JÜLICH
Forschungszentrum

FUNDAMENTAL INTERACTIONS – QED

Basics

Electromagnetic processes – overview

Electromagnetic phenomena and reactions can be arranged according to their complexity:

- Characteristics of charged leptons: **anomalous magnetic moments**
- Processes involving charged leptons only: **Bremsstrahlung**, **Moller** and **Bhabha scattering**, **lepton annihilation** and **pair production**,
- Reactions of charged leptons with hadrons and nuclei: **(exotic) atoms, elastic** and **inelastic electron scattering**
- Reactions of charged hadrons/nuclei: **Coulomb scattering**

It turns out that (due to “loops”), even the most elementary processes are complex (and influenced by the other fundamental interactions)

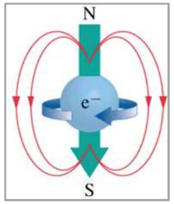
FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – leptonic anomalous magnetic moments (I)

The Dirac equation predicts a **magnetic dipole moment** of a particle with spin-½ to be:

$$\mu_S = -g_S \frac{e}{2m} S$$



with the so called (spin) "gyromagnetic ratio" (g-factor) $\boldsymbol{g_s = 2}$.

The **experimental results** for the **electron** and **muon** show a small difference from 2, measured with fantastic precision and parametrized by the **anomalous magnetic moment**: $a = ½ (g – 2)$:

$a_e = (11596521.8091 \pm 0.0026) \times 10^{-10}$ $(g_e/2 = 1.0011 \ldots)$

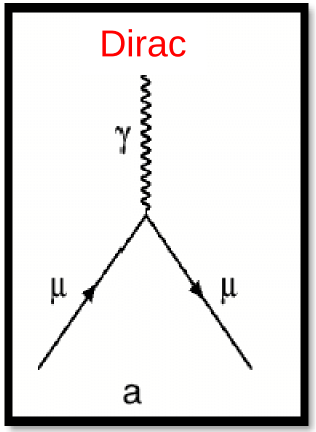
$a_\mu = (11659209 \pm 6) \times 10^{-10}$

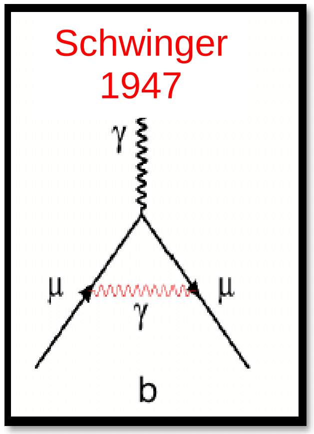
FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – leptonic anomalous magnetic moments (II)

Quantum loop effects lead to this small, extremely precisely calculable deviations from 2;



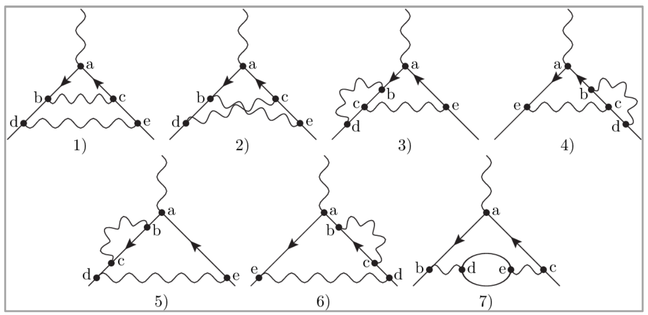


most important one:

$$a_{\mu} = \frac{\alpha}{2\pi} \approx \frac{1}{800}$$

=0.001614

Higher order loops:



FUNDAMENTAL INTERACTIONS – QED

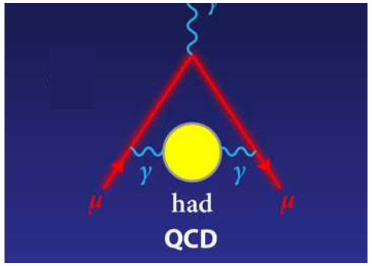
Examples

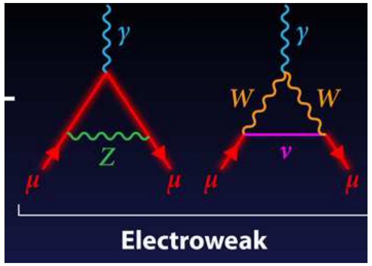
Electromagnetic processes – leptonic anomalous magnetic moments (III)

The QED prediction agrees with the experimentally measured value to more than 10 significant figures, making the magnetic moments the **most accurately verified prediction** in the history of physics

There are two major issues as of today:

- Besides QED there are **contributions from the strong** and **weak IA**:





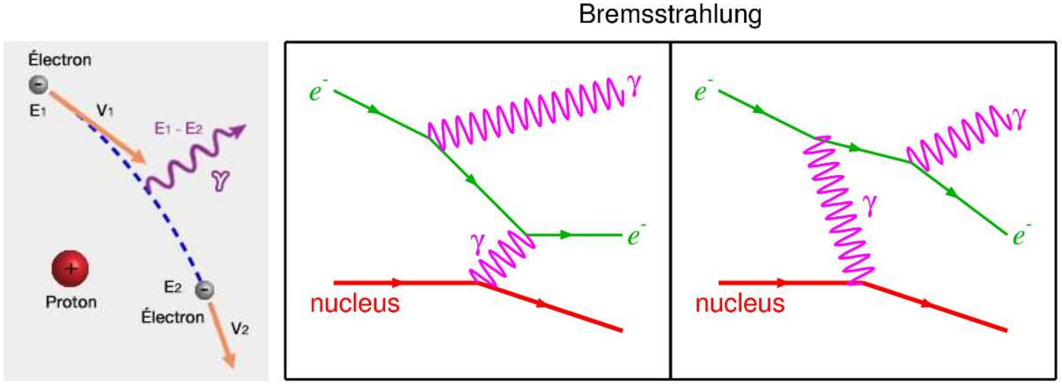
- The best/most complete calculations leave a **discrepancy** (~ 3σ, 95%) between experiment and theory for the muon → new physics? But first: new measurement!

FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – bremsstrahlung (I)

"**Bremsstrahlung**" is electromagnetic radiation (photons) produced by the deceleration of a charged particle (mostly electron) when deflected by another charged particle, typically an atomic nucleus:



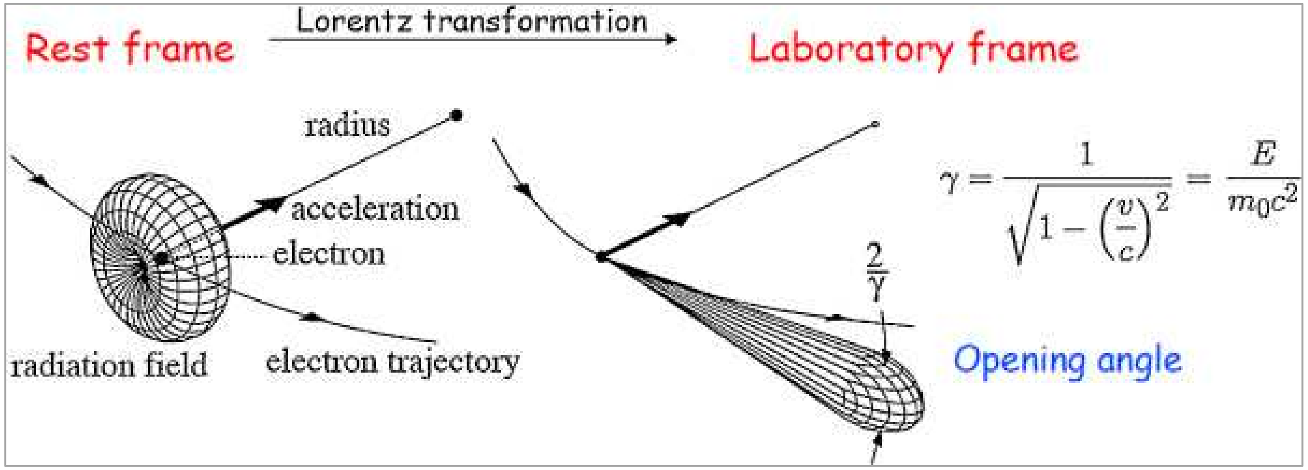
The complete quantum mechanical description was first performed by **Bethe** and **Heitler** (1935): $\sigma \sim Z^2\alpha^3/m_e^2$ … modern improvements

FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – bremsstrahlung (II)

Since bremsstrahlung is often emitted by high-energy electrons, the dipole pattern in the **rest frame** is “boosted” into a small forward cone in the **laboratory frame**:



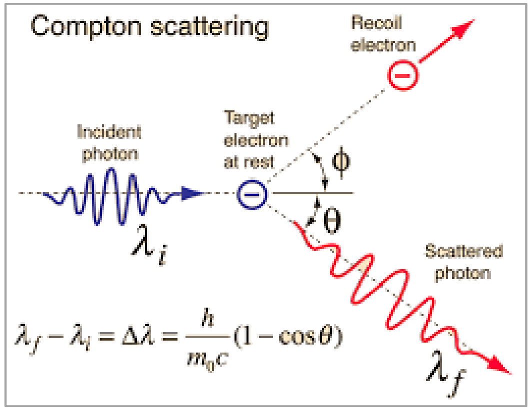
→ **Synchrotron radiation** in circular electron accelerators/storage rings

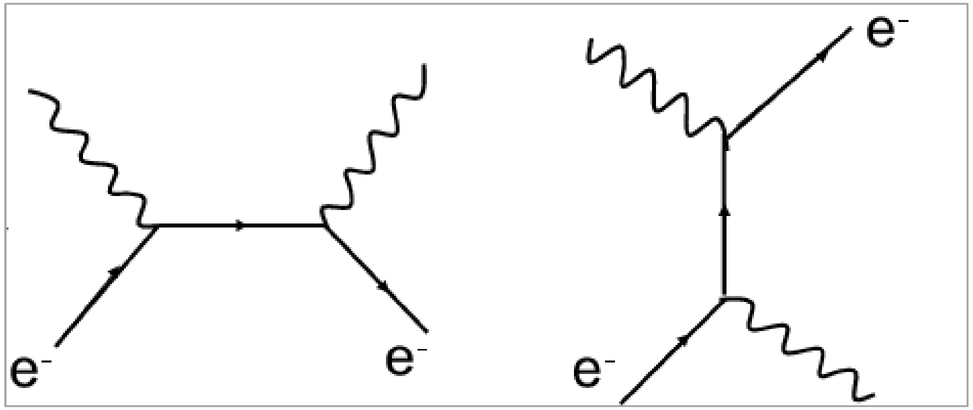
FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – Compton scattering

"**Compton scattering**" (Arthur Holly Compton, 1925) is the scattering of a **photon by a charged particle**, usually an electron; part of the energy of the photon is transferred to the recoiling electron (→ **Compton effect**):





Cross section is the **Klein-Nishina formula** (1929) …

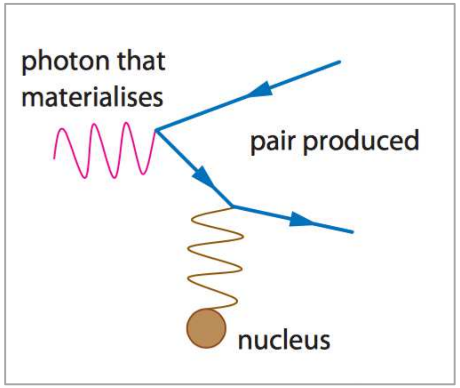
Inverse Compton scattering → e.g. high energy γ´s (Compton backscatt.)

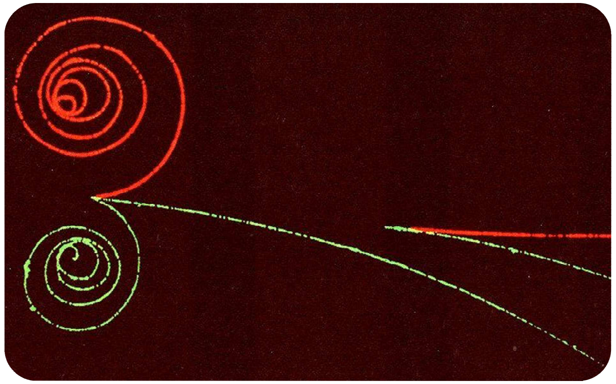
FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – pair production

"**Pair production**" is the creation particle-antiparticle pair from a neutral boson. It often refers specifically to a **photon creating an electron–positron pair** near a nucleus. The incoming γ-energy must be above a threshold of at least the total rest mass energy of the two particles, the reaction must **conserve** both **energy and momentum**:



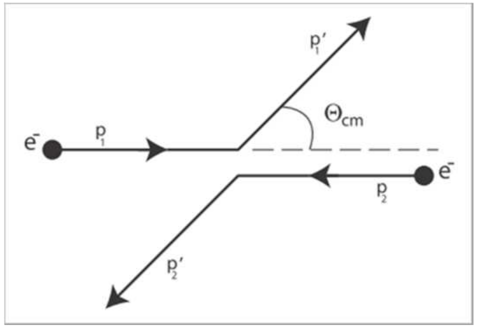


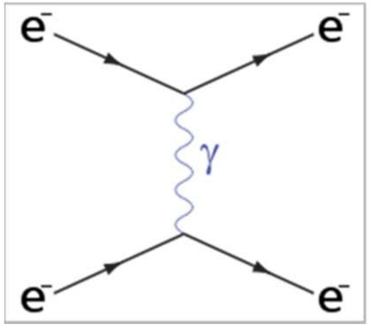
FUNDAMENTAL INTERACTIONS – QED

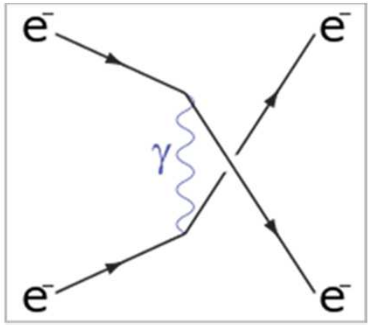
Examples

Electromagnetic processes – charged lepton scattering (I)

"**Møller scattering**" (Christian Møller) is electron-electron scattering: there are two lowest-order Feynman diagrams describing the process:







Note: two indistinguishable particles

Cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8E^2}\left[\frac{1+\cos^4(\theta/2)}{\sin^4(\theta/2)} + \frac{2}{\sin^2(\theta/2)\cos^2(\theta/2)} + \frac{1+\sin^4(\theta/2)}{\cos^4(\theta/2)}\right]$$

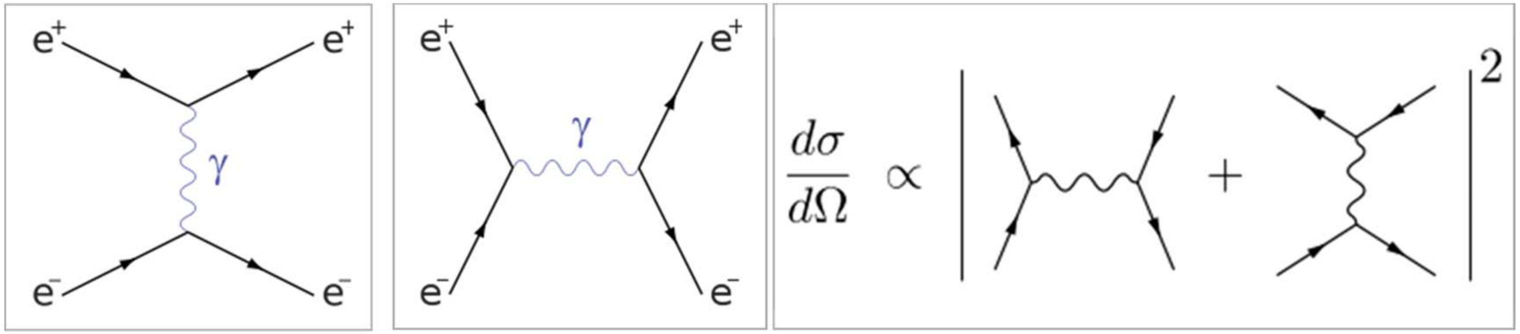
Many more diagrams (loops) … including weak interaction (Z-exchange)!

FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – charged lepton scattering (II)

“**Bhabha scattering**” (Homi J. Babha) is the electron-positron scattering process; there are two QED diagrams (lowest order) contributing: **scattering** and **annihilation**:



Cross section:

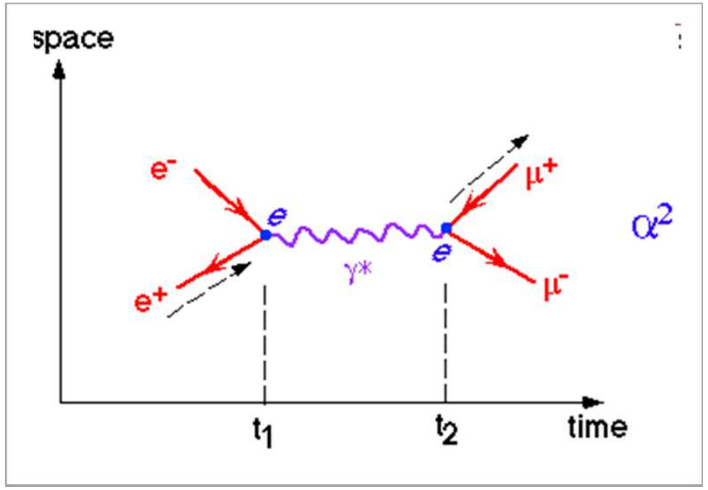
$$\frac{d\sigma}{d\Omega} = \frac{e^4}{32\pi^2 E_{\text{cm}}^2}\left[\frac{1+\cos^4\frac{\theta}{2}}{\sin^4\frac{\theta}{2}} - \frac{2\cos^4\frac{\theta}{2}}{\sin^2\frac{\theta}{2}} + \frac{1+\cos^2\theta}{2}\right]$$

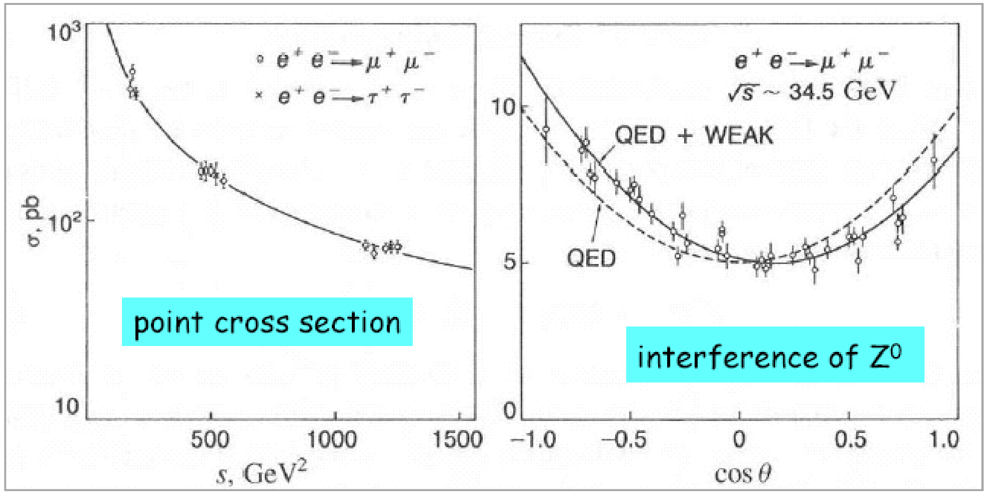
FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – lepton pair production

If the energy of the virtual annihilation photon is high enough, a higher mass lepton pair (e.g. $\mu^+\mu^-$) can be produced:





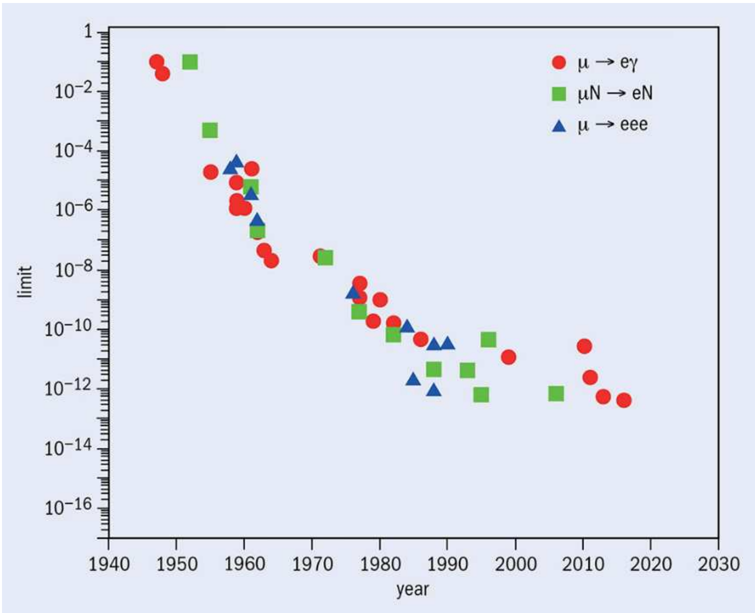
Point cross section fits data (no sign of internal lepton structure); angular distribution indicates asymmetry → weak interaction contribution

FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – radiative lepton decays

The processes $\mu \rightarrow$ **e** γ or $\mu \rightarrow$ **ee^+e^-** (such that the electric charge is conserved) have been search for in experiments (e.g. MEG, Mu3e), but they have never been observed; ever improving **upper limits**:



Note: decays would be **lepton-flavor violating**

JÜLICH Forschungszentrum

FUNDAMENTAL INTERACTIONS – QED

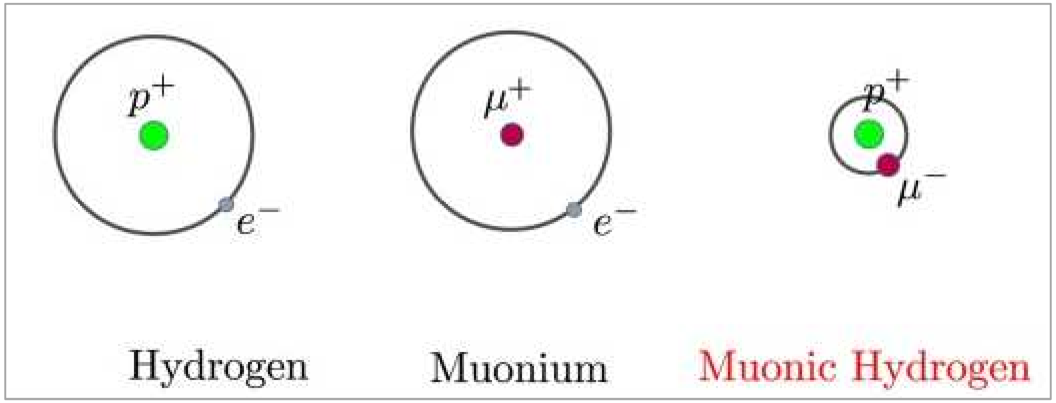
Examples

Electromagnetic processes – exotic leptonic atoms

Consider the hydrogen atom (pe bound system): exchanging the electron by a muon leads to "**muonic hydrogen**" (pμ)

If the proton is replaced by a muon, the corresponding bound (μe)-system is called "**muonium**"

Note: Bohr radius $r_{H,e} \sim 2.59 \times 10^{-11}$ m



Note: Bohr radius $r_{H,\mu} \sim 2.56 \times 10^{-13}$ m

Precise QED calculations: energy levels, spectroscopy (e.g. Lamb shift)

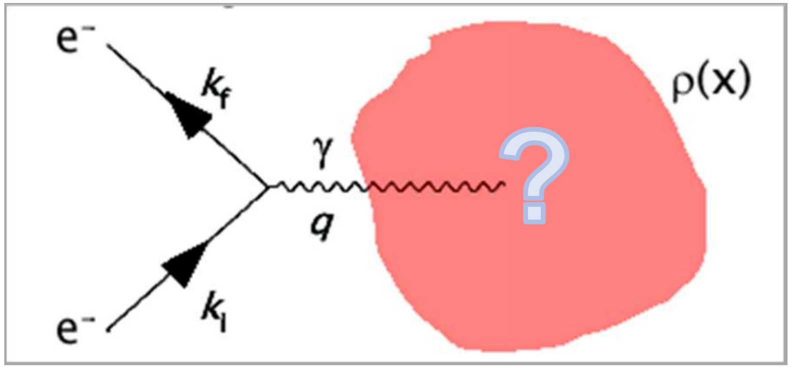
Influence of the **nuclear** charge distribution

FUNDAMENTAL INTERACTIONS – QED

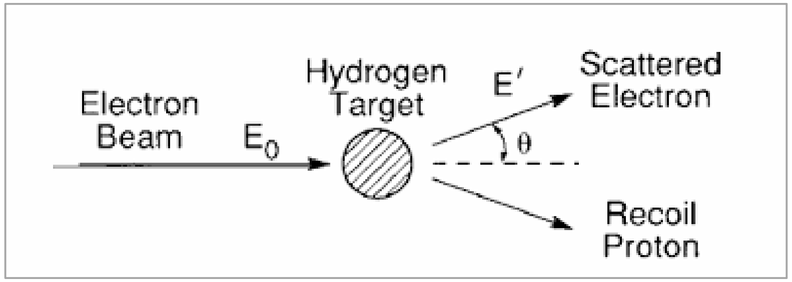
Examples

Electromagnetic processes – electron scattering (I)

The well-known **electromagnetic interaction** can be used as a **probe** to investigate other objects, e.g. electron scattering on nucleons and nuclei:



→ **R. Hofstadter**



Nobel prize1961

FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – electron scattering (II): cross section

The scattering of **relativistic electrons** (E >> me) by a charge distribution can be calculated using standard methods of quantum mechanics:

(i) If the electron were spinless and scattered from a static point charge, the cross section would be given by the **Rutherford formula**:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{1}{2}\theta}$$

(ii) Taking into account the electron´s spin gives the "**Mott cross section**":

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2 \frac{1}{2}\theta}{4E^2 \sin^4 \frac{1}{2}\theta}$$

FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – electron scattering (III): cross section

(iii) The elastic scattering of an electron by a **point-like Dirac particle** of mass M has the cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2 \frac{1}{2}\theta}{4E^2 \sin^4 \frac{1}{2}\theta} \cdot \frac{E'}{E} \left[1 - \frac{q^2}{2M^2} \tan^2 \frac{1}{2}\theta \right]$$

(Note: reduces to the Mott cross section for increasing M)

(iv) These results do not apply for an extended charge distribution ρ(r); here the scattering amplitude is modified by a **form factor $F(q^2)$**:

$$F(q^2) = \int d^3 r e^{i\mathbf{q}\cdot\mathbf{r}} \rho(r)$$

so the cross sections are multiplied by a factor $|F(q^2)|^2$

FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – electron scattering (IV): cross section

(v) Finally the differential cross section for elastic electron proton scattering is given in terms of the so called **Rosenbluth formula** with two **form factors F_1** and **F_2** (θ lab scattering angle; κ anomalous magnetic moment; $Q^2 = - q^2$):

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2 \frac{1}{2}\theta}{4E^2 \sin^4 \frac{1}{2}\theta} \cdot \frac{E'}{E} \cdot \left[\left(F_1^2 + \frac{\kappa^2 Q^2}{4M^2} F_2^2 \right) + \frac{Q^2}{2M^2} (F_1 + \kappa F_2)^2 \tan^2 \frac{1}{2}\theta \right]$$

Note: if the proton were a point-like particle like the electron, $F_1 = 1$; $\kappa F_2 = 0$

Experimentally, the form factor is determined from the ratio:

$$[F(\mathrm{q})]^2 = \frac{\sigma(\mathrm{q})}{\sigma_{\mathrm{Mott}}(\mathrm{q})}, \quad \leftrightarrow \quad |\text{Form factor}|^2 = \frac{\sigma(\text{structured object})}{\sigma(\text{pointlike object})}$$

FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – electron scattering (V): form factor

The nucleon (proton and neutron) **electromagnetic form factors** describe the **spatial distributions of electric charge and current** inside the nucleon and thus are intimately related to its internal structure; an expansion (for small q^2) gives:

$$\begin{aligned} F(q^2) &= \int d^3r \rho(r) \exp(i\mathbf{q}\cdot\mathbf{r}) \\ &= \int d^3r \rho(r)[1 + i\mathbf{q}\cdot\mathbf{r} - (1/2)(\mathbf{q}\cdot\mathbf{r})^2 \cdots] \\ &= 1 - \frac{\mathbf{q}^2}{6} < r^2 > \cdots \end{aligned}$$

→ $<r^2>$ is the so called "**root-mean-square radius**" (r_{rms})

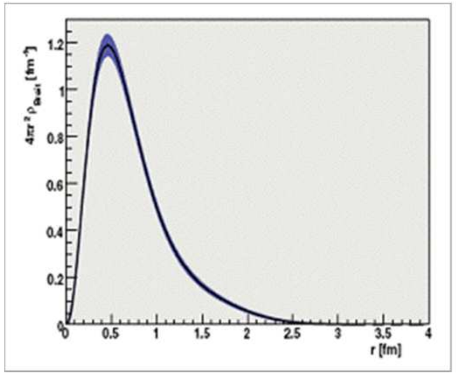
FUNDAMENTAL INTERACTIONS – QED

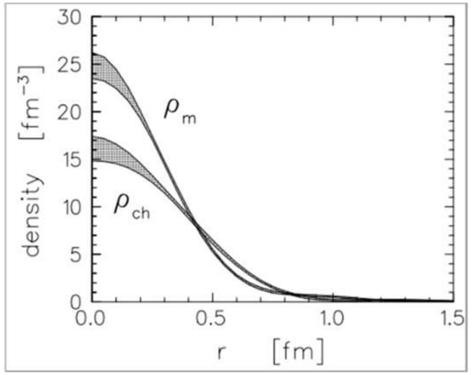
Examples

Electromagnetic processes – electron scattering (VI): results

Results: charge and magnetization distributions

Proton:

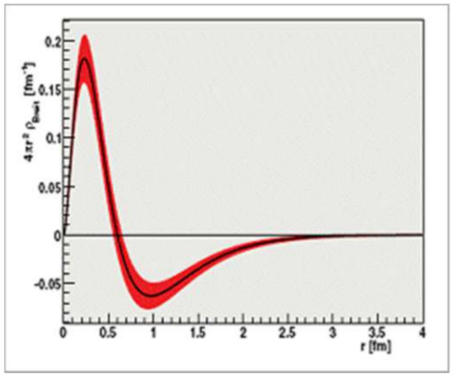


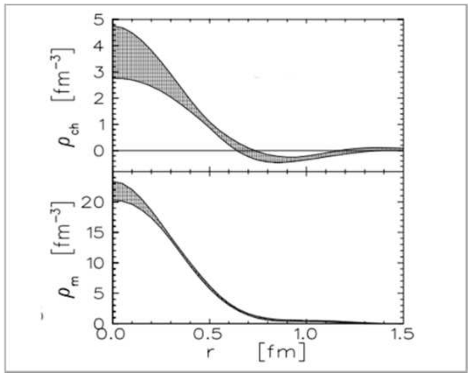


r_{rms}~ **0.88 fm**

Note: proton radius from μ-hydrogen is about 0.84 fm

Neutron:





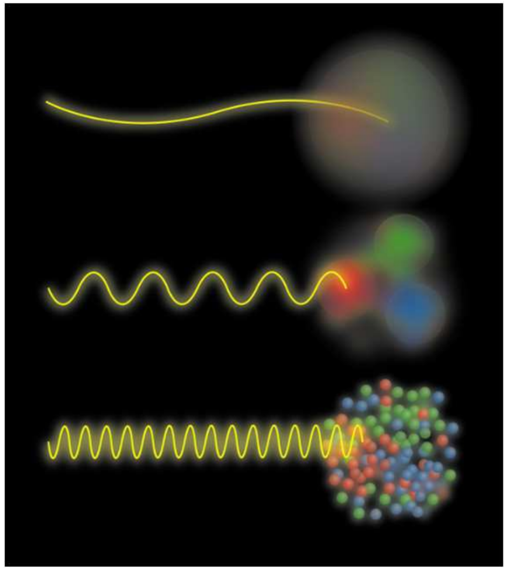
JÜLICH Forschungszentrum

FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – electron scattering (VII): outlook

Besides **elastic electron scattering**, **inelastic** (including the excitation of nucleon resonances) and **deep inelastic electron scattering** (DIS) are studied – depends on energy (wavelength of (virtual photon); will be discussed in "QCD section":

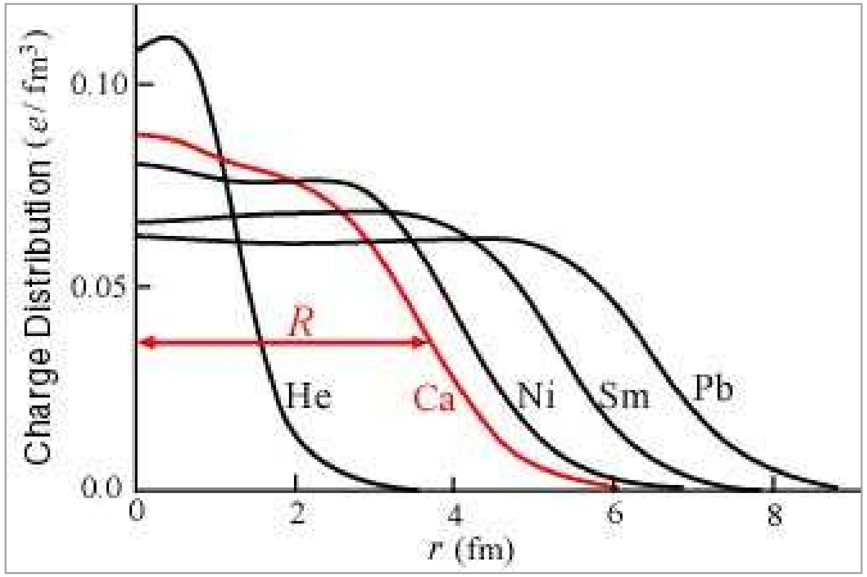


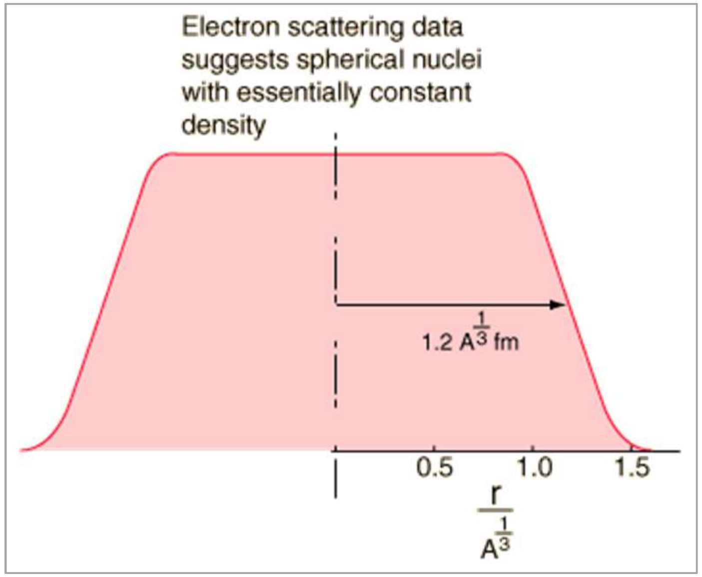
FUNDAMENTAL INTERACTIONS – QED

Examples

Electromagnetic processes – electron scattering (VIII): nuclei

Elastic electron scattering on **nuclei** reveals the **charge distribution**:





→ Central density, surface thickness, scaling of size …

FUNDAMENTAL INTERACTIONS – QED

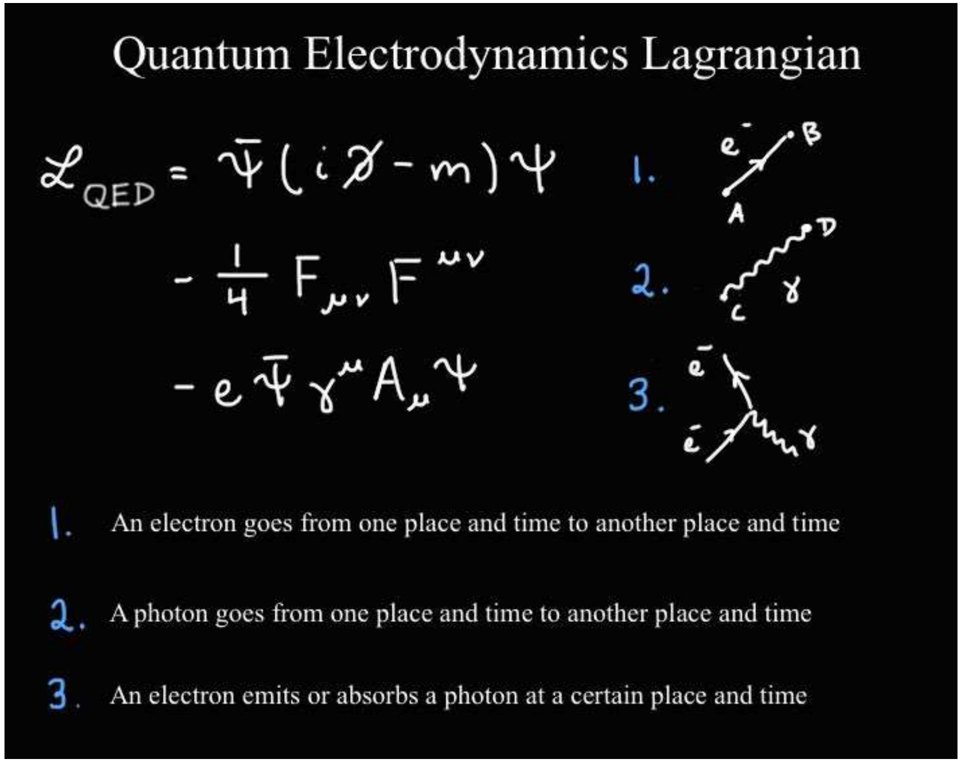
Examples

Electromagnetic processes – (…)

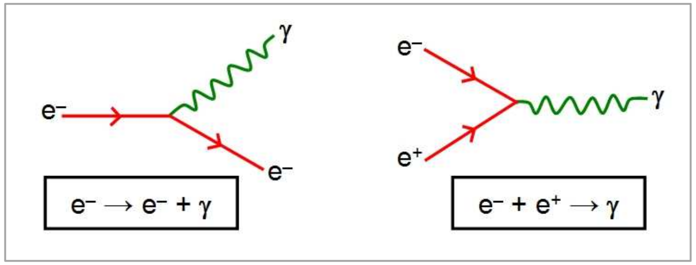
FUNDAMENTAL INTERACTIONS – QED

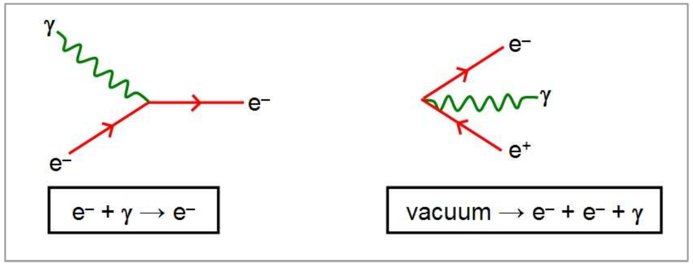
Summary

Quantum Electrodynamics (QED)



→ elementary "vertices":





– must be combined for real processes …

THE FORCES

That´s it for today